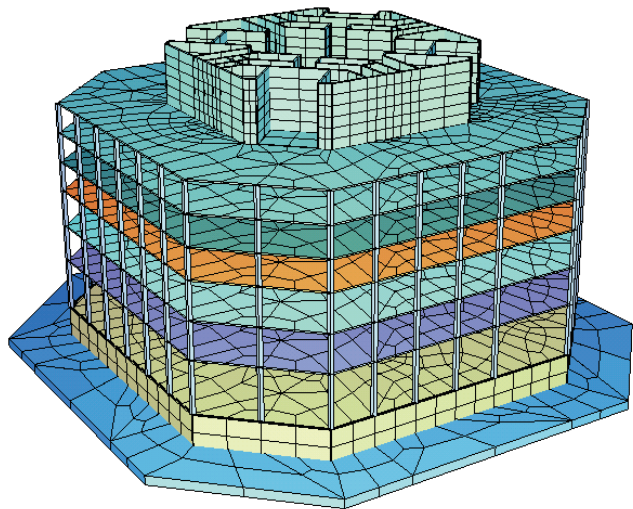


Sensitivitätsanalyse mit Einflussfunktionen

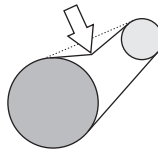
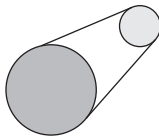
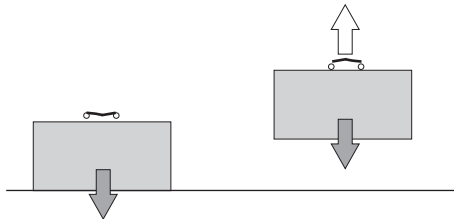
Institut für Statik und Dynamik, Universität Kassel

January 21, 2011

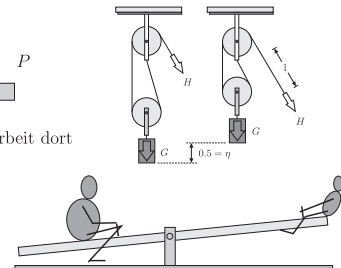
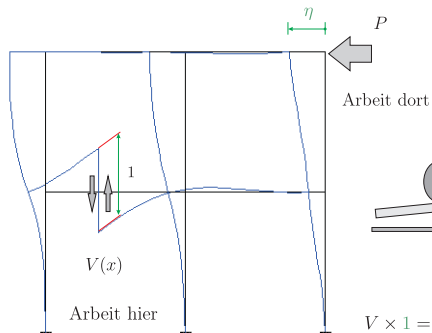
Statik mit finiten Elementen



Messen (Bemessen) ist Bewegen



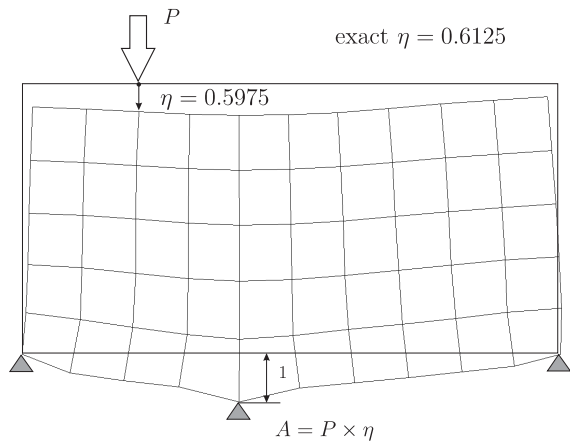
Statik ist angewandte Kinematik



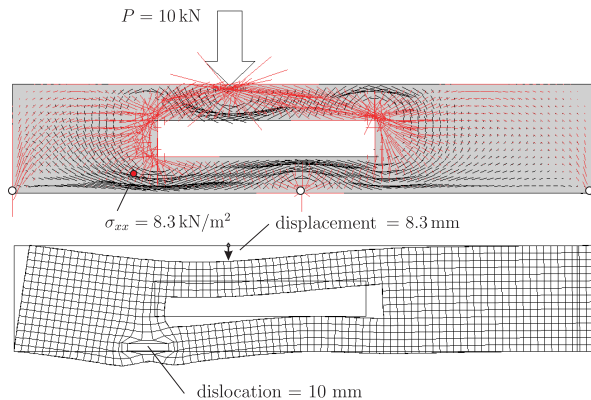
$$V \times 1 = P \times \eta$$

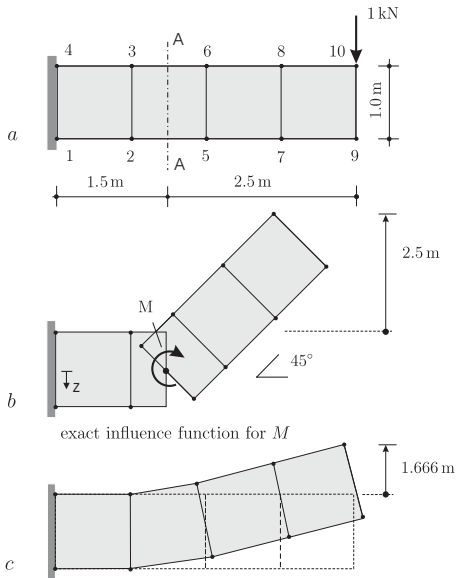
Arbeit hier = Arbeit dort

Finite Elemente = eingeschränkte Kinematik



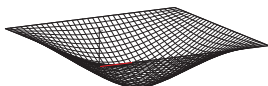
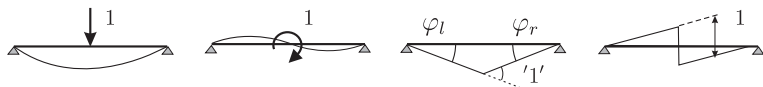
Ungenauere Einflussfunktionen



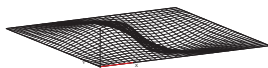


FE-approximation on V_h (= bilinear elements)

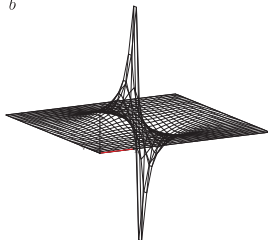
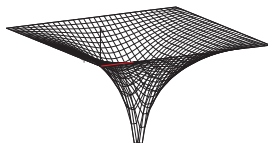
Die Natur der Einflussfunktionen

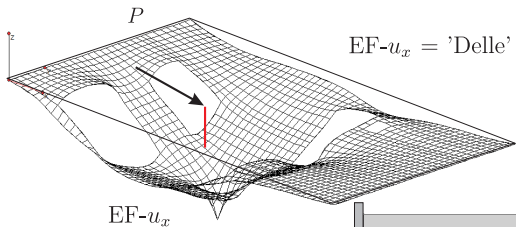


a



b

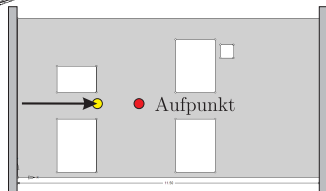




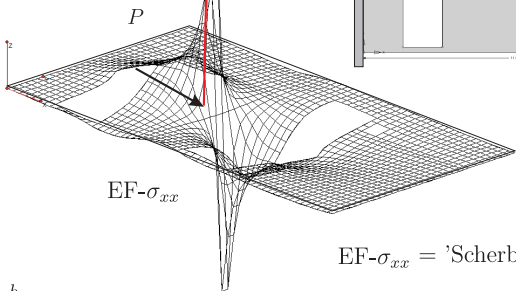
a

Last P

Aufpunkt

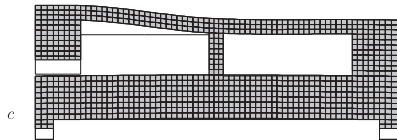
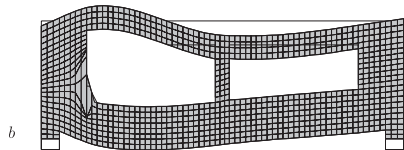
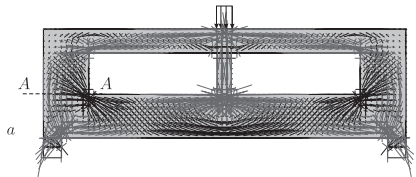


c

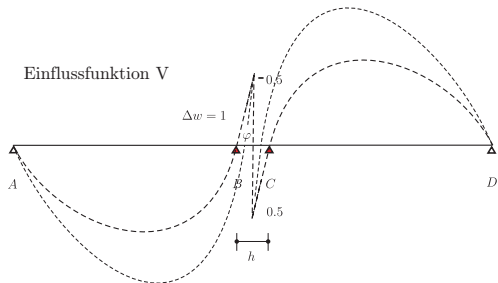
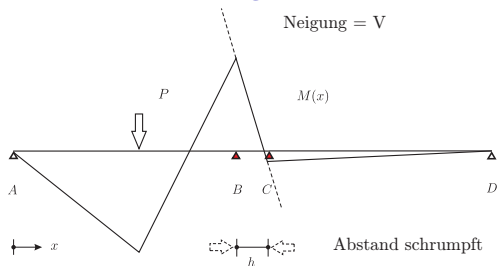


b

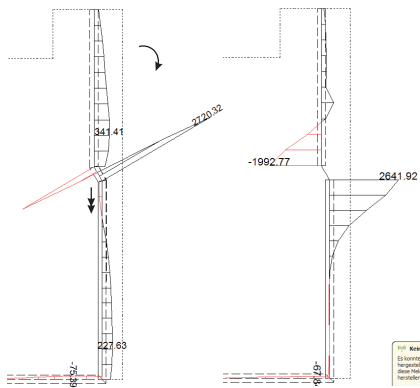
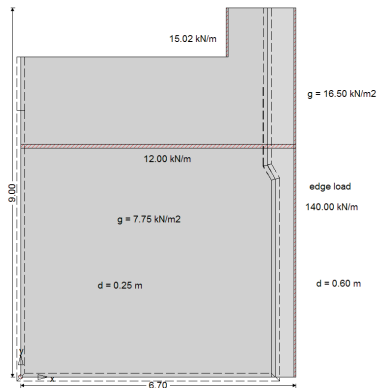
Singularitäten



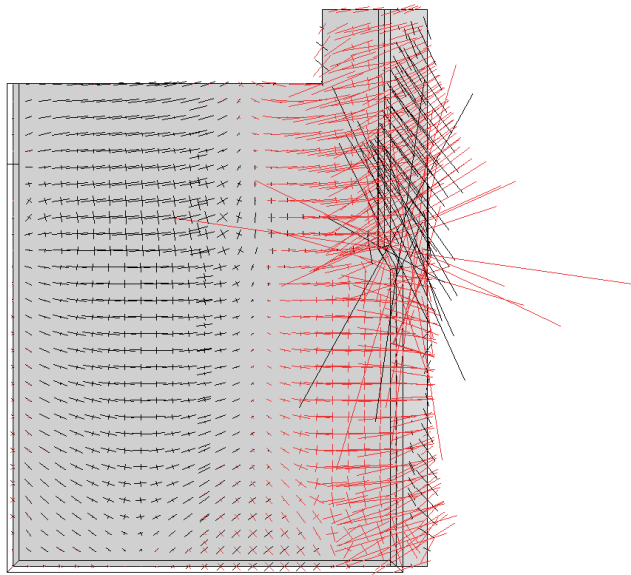
V wird unendlich groß



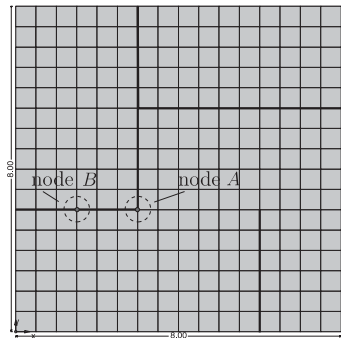
Keine Theorie



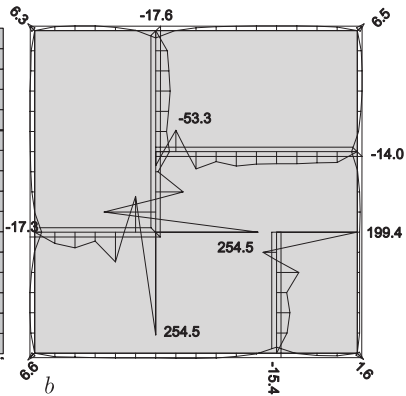
Hauptmomente



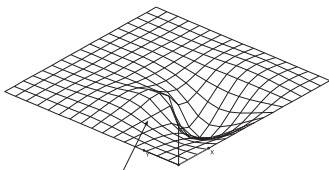
Lagerkräfte



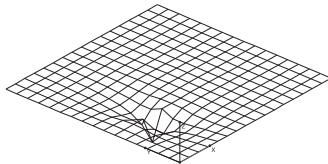
a



b

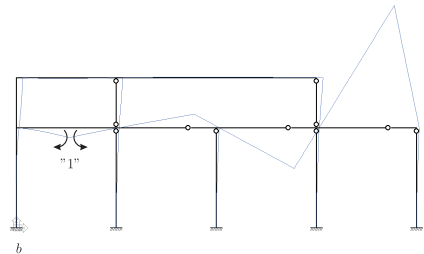
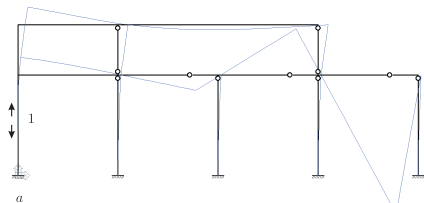


wall $w = 0$ except a node A

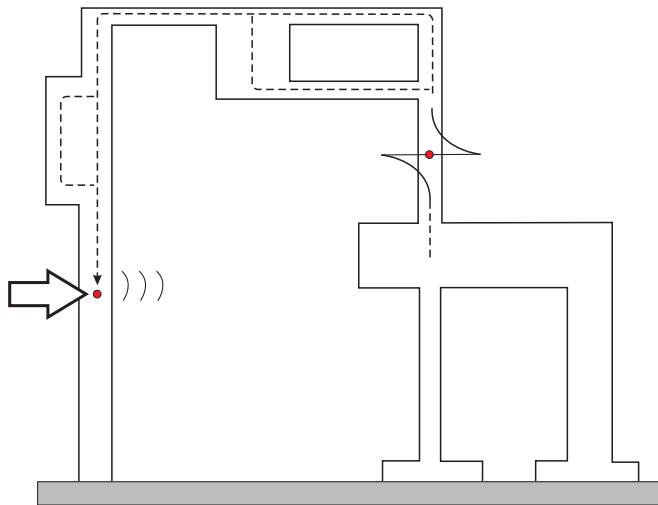


d

Handle with care...



Biege- und Diskontinuitätsbereiche



Wie robust ist das Tragwerk?

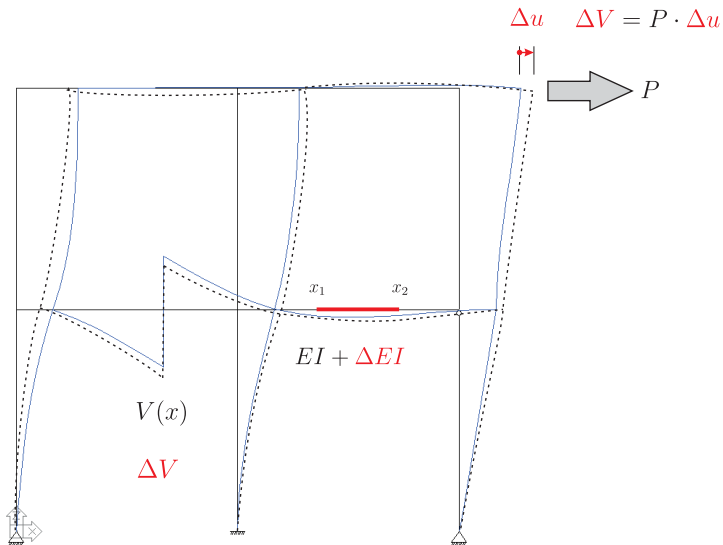
Welche Störungen bewirken große
Änderungen?

Alles ändert sich, sobald sich nur eine Zahl ändert.

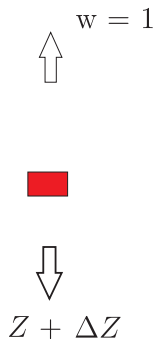
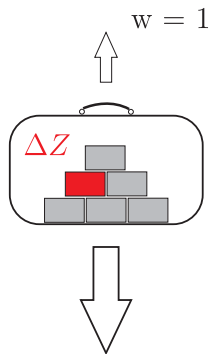
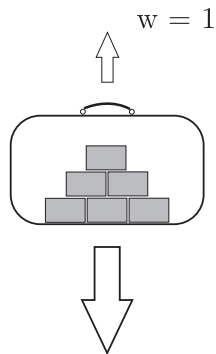
$$\mathbf{K} = \frac{1}{5} \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}, \Rightarrow \mathbf{K}^{-1} = \begin{bmatrix} 4.00 & 3.00 & 2.00 & 1.00 \\ 3.00 & 6.00 & 4.00 & 2.00 \\ 2.00 & 4.00 & 6.00 & 3.00 \\ 1.00 & 2.00 & 3.00 & 4.00 \end{bmatrix},$$

$$\mathbf{K} = \frac{1}{5} \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2.1 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}, \Rightarrow \mathbf{K}^{-1} = \begin{bmatrix} 3.83 & -2.68 & 1.78 & -0.89 \\ -2.68 & 5.36 & -3.57 & 1.79 \\ 1.79 & -3.57 & 5.71 & -2.86 \\ -0.89 & 1.78 & -2.86 & 3.93 \end{bmatrix},$$

$$\mathbf{u} = \mathbf{K}^{-1} \mathbf{f}$$



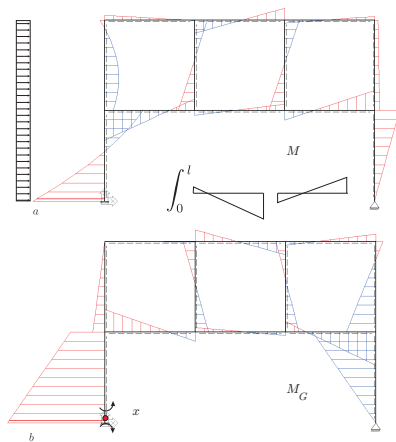
Lokale Analyse



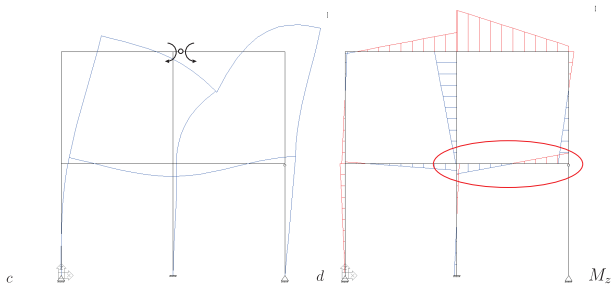
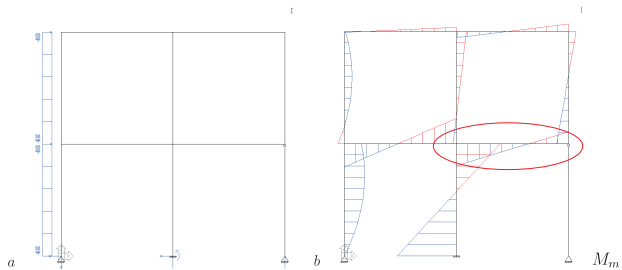
$$\Delta G = \Delta Z \cdot 1$$

$$\Delta G = G_{neu} - G = \Delta Z$$

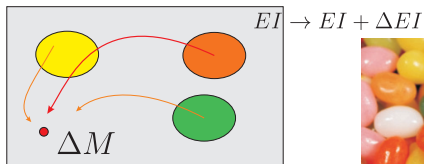
Die neue Formel



$$\Delta M = \frac{\Delta EI}{EI} \int_0^l \frac{M M_G}{EI} dx$$



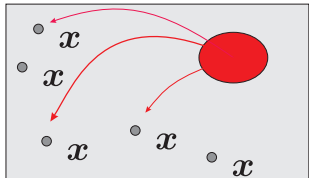
Fokus auf einen Punkt



$$\Delta M = \frac{\Delta EI}{EI} \int_0^l \frac{M M_G}{EI} dx$$



Wie weit strahlen Änderungen aus?



Wo ist der Trick?

$$\Delta M = \frac{\Delta EI}{EI} \int_0^l \frac{M \bar{M}^{Neu}}{EI} dx$$

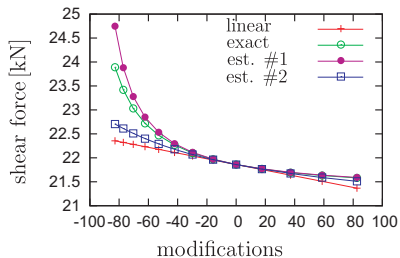
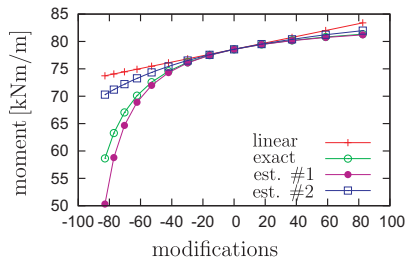
← neue Einflussfunktion

$$\Delta M = \frac{\Delta EI}{EI} \int_0^l \frac{M \bar{M}}{EI} dx + \underbrace{\frac{\Delta EI}{EI} \int_0^l \frac{M (\bar{M}^{Neu} - \bar{M})}{EI} dx}_{\sim 0}$$

$$\Delta M = \frac{\Delta EI}{EI} \int_0^l \frac{M \bar{M}}{EI} dx$$

← alte Einflussfunktion

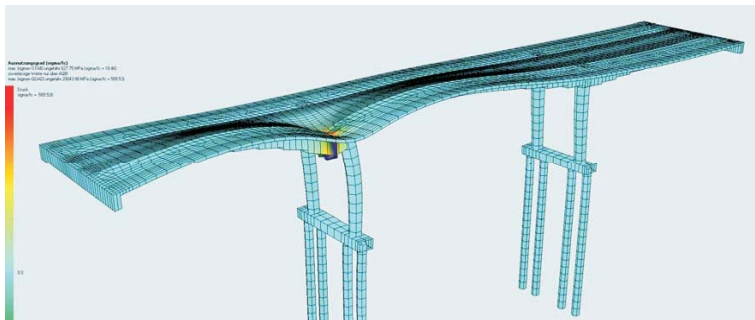
Linearisierung

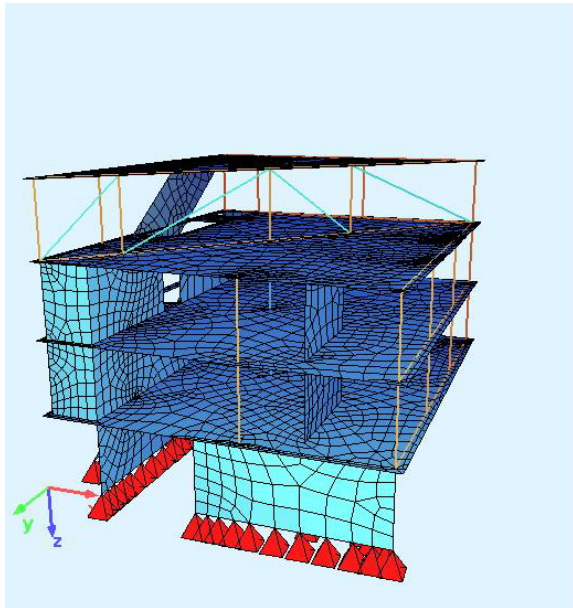


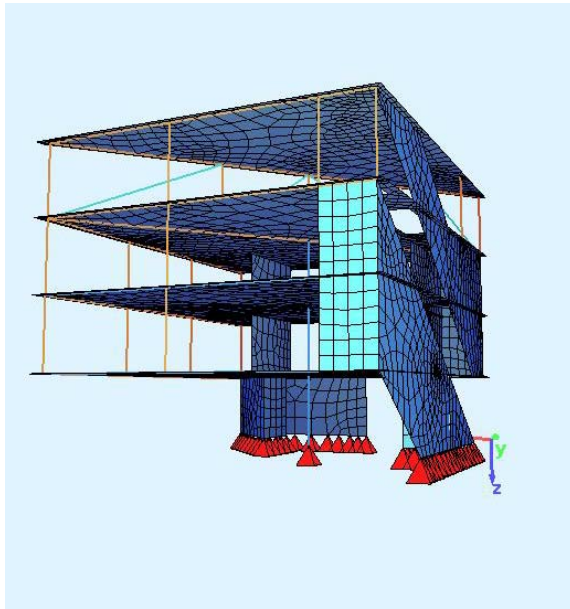
Semi-integrale Brücke

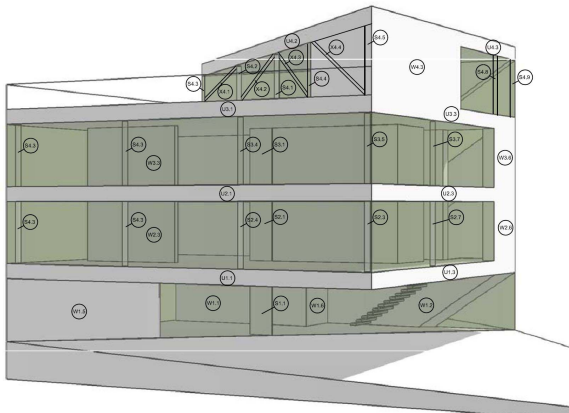


Einflussfunktion für M







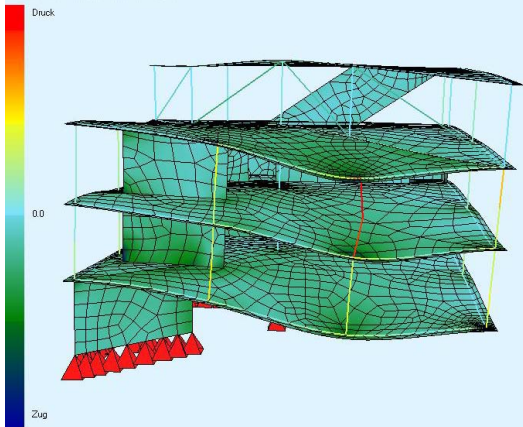


Ausnutzungsgrad (σ/σ_c)

max. | $\sigma_{\text{max-STAB}}$ | ungefähr 97573.69 MPa

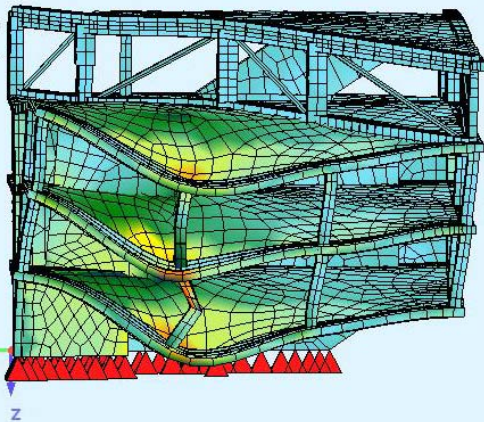
zuverlässige Werte nur über AQ8!

max. | $\sigma_{\text{max-QUAD}}$ | ungefähr 6329.26 MPa

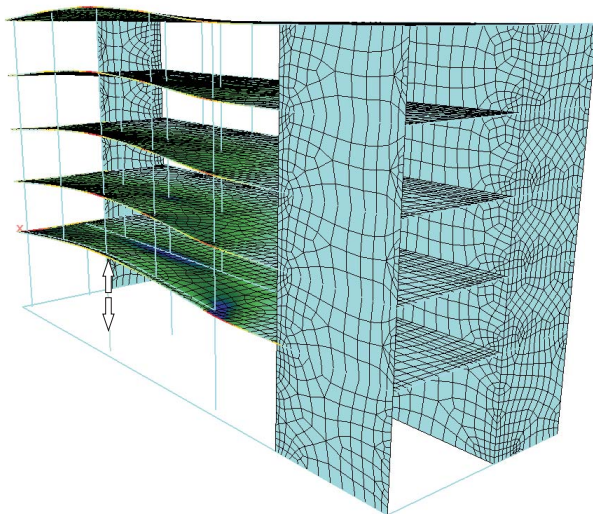


1,43 MPa

3,38 MPa



Bürogebäude

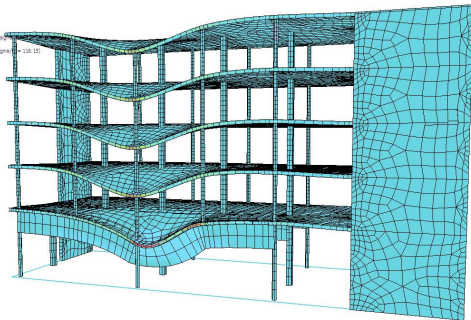


Asmutsungsgrad (sigma/fc)
neu: sigma = -0,031 ungefähr 62,090, 30 MPa (50%)
zulässige (Verte nur über ACE)
neu: sigma = -0,040 ungefähr 2362,87 MPa (sigma = 128,13)

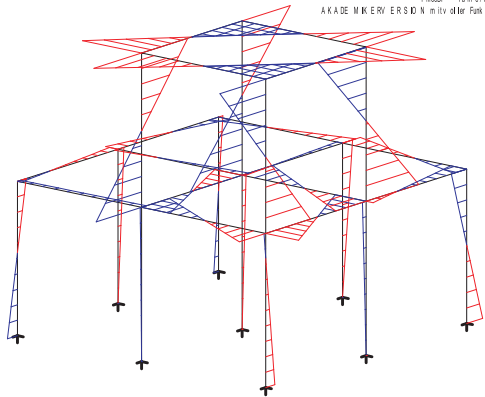
Druck
sigma/fc = -0,0204, 516



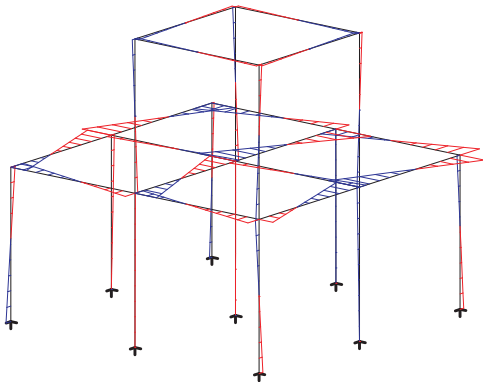
Zug
sigma/fc = 0,0104, 516



- [1] Sopoht M, Sopoht G (2008) Sensitivitätsanalyse an einem Brückenbauwerk in semi-integraler Bauweise, Diplomarbeit Universität Kassel
- [2] Takunov B (2009) Sensitivitätsanalyse an dem FE-Modell eines Bürogebäudes, Diplomarbeit Universität Kassel
- [3] Otaibi W (2009) Sensitivitätsanalyse an ebenen und räumlichen Rahmen, Diplomarbeit Universität Kassel
- [4] Otaibi W (2010) FMEA (Failure Mode and Effects Analysis) eines Tragwerks, Masterarbeit Universität Kassel
- [5] Otaibi W, Takunov B (2010) Überführung eines 3D- in ein 2D-Modell, Projektarbeit Universität Kassel
- [6] Takunov B (2010) Sensitivitätsanalyse mittels Einflussfunktionen, Masterarbeit Universität Kassel
- [7] Kunow, T (2009) Modellfehler und Greensche Funktionen in der Statik, Dissertation Universität Kassel
- [8] Carl, O (2010) Statische und dynamische Sensitivitätsanalysen von geschädigten Tragwerken mit Greenschen Funktionen, Promotion, Universität Siegen
- [9] Hartmann F, Katz C (2007) Structural Analysis with Finite Elements, Springer Verlag, Berlin Heidelberg New York, 2. Auflage

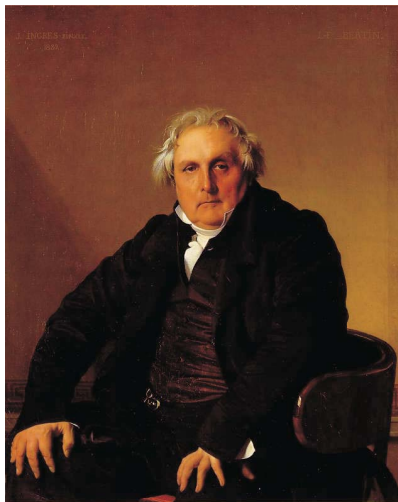


Position: System 2	Målestab: 1/75
Mzz [kNm] - L004	
Titel: Load och Stödb...	

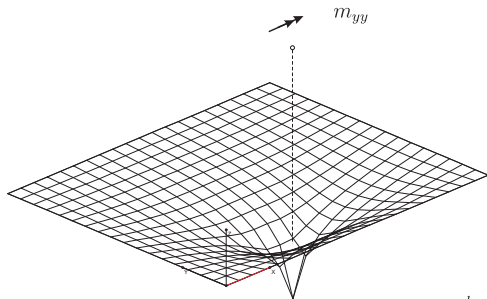


Position: System 2	
Mzz [kNm] - LC001	Maßstab: 1:75
Titel: LO rd, o hreeS chubw...	

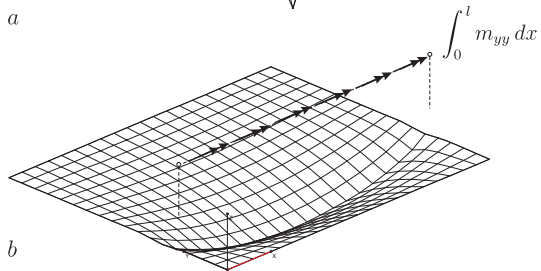
FE = limited kinematics



Point values and integral values

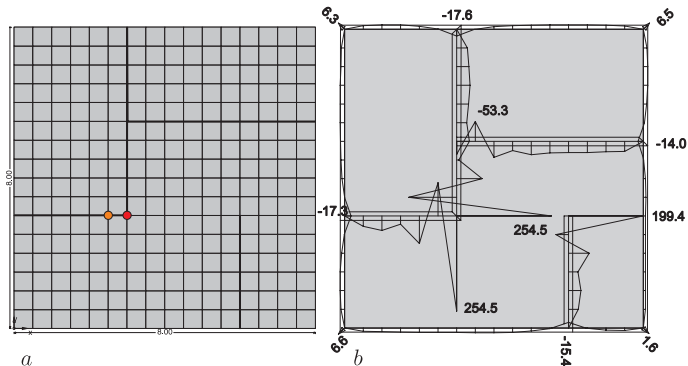


a

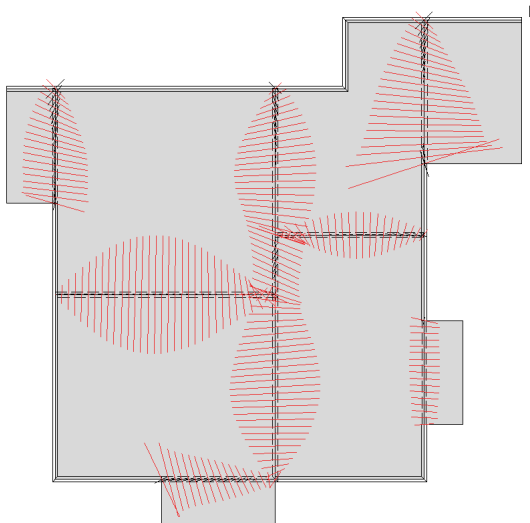


b

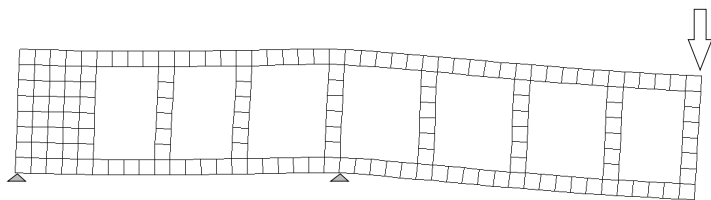
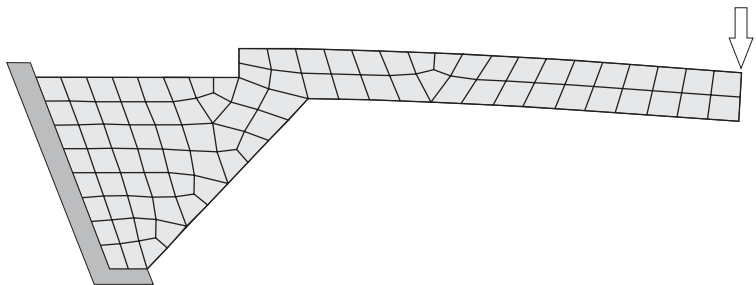
Oszillierende Lagerkräfte müssen nicht falsch sein.



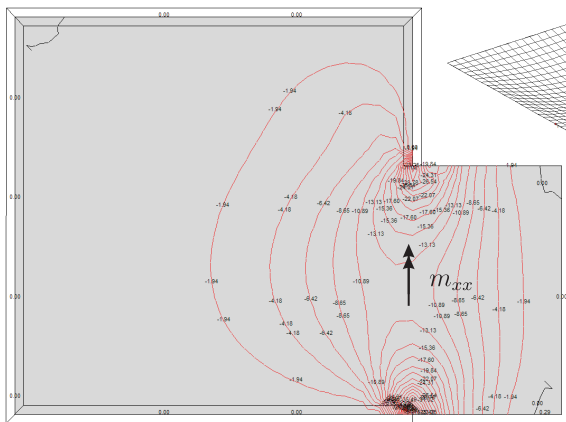
Biegemomente abrunden?



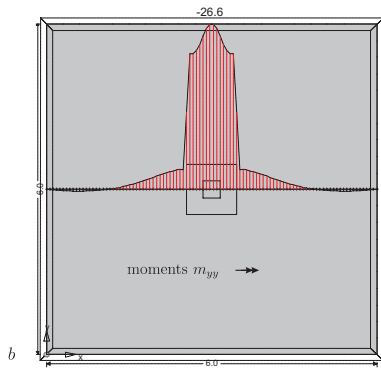
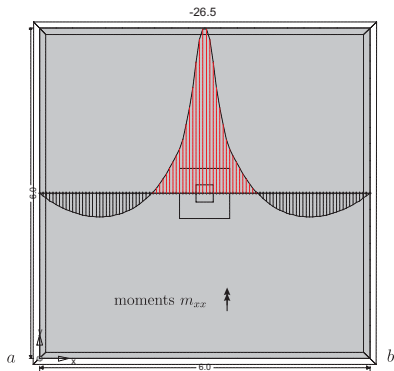
Vorsicht bei labilen Strukturen



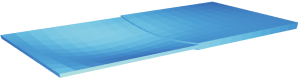
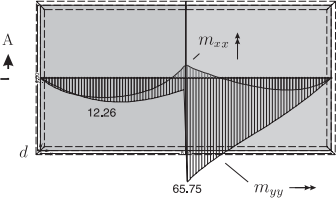
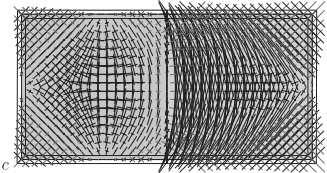
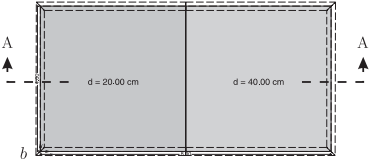
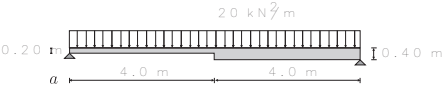
Singuläre Punkte müssen verfeinert werden.



Stützenkopfverstärkungen



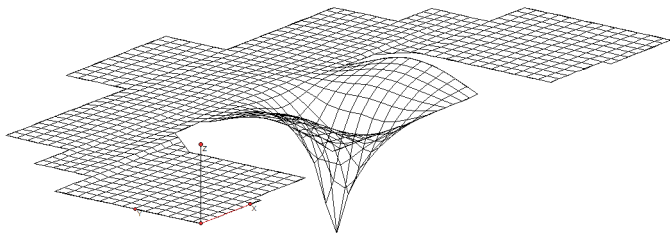
Unterschiedliche Deckenstärken



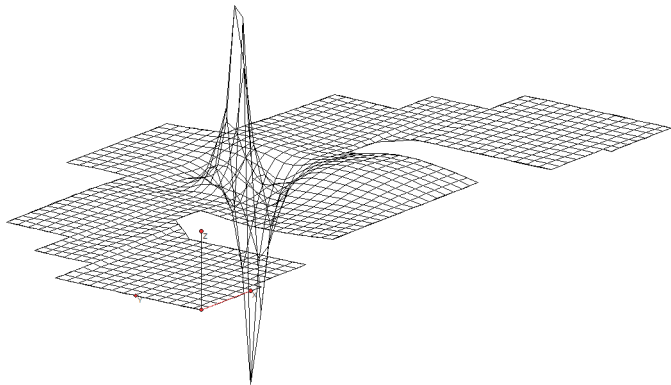
$$u_y(x) - u_y^h(x) = \int_{\Omega} [G_0(y, x) (\delta_M[y] - \delta_M^h[y])] d\Omega_y = a(G_0, G_M - G_M^h) \quad (1)$$

$$\begin{aligned} |a(G_0, G_M - G_M^h)| &\leq a(G_0, G_0)^{1/2} \cdot a(G_M - G_M^h, G_M - G_M^h)^{1/2} \\ &= \|G_0\|_E \cdot \|G_M - G_M^h\|_E. \end{aligned} \quad (2)$$

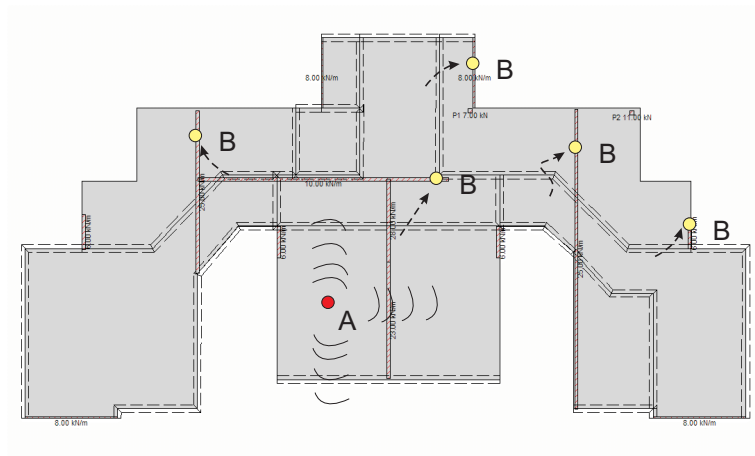
Moment m_x



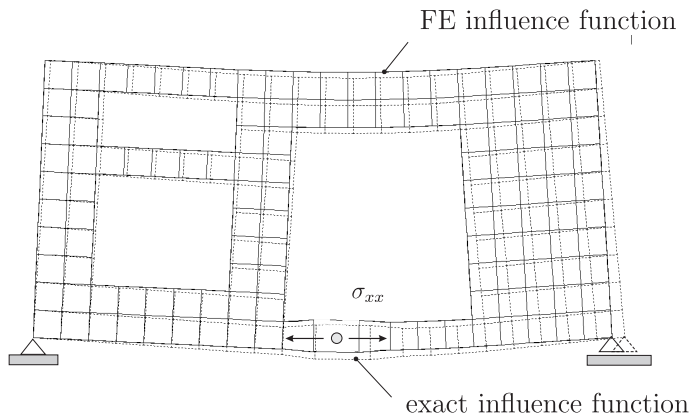
Shear force v_x



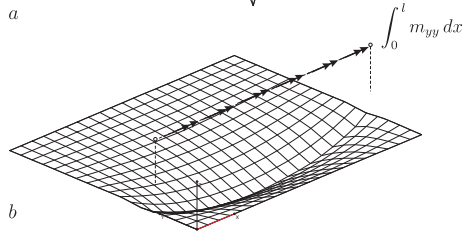
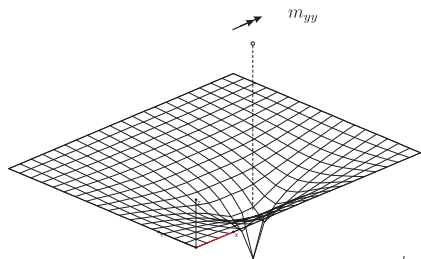
What is sensed in the distance and how accurate is it?



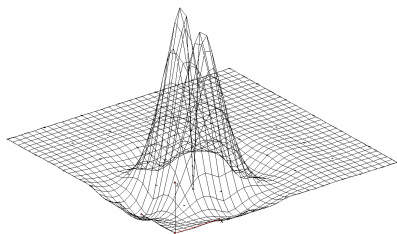
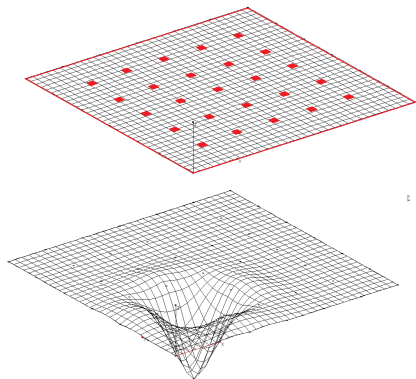
FE = (limited) kinematics



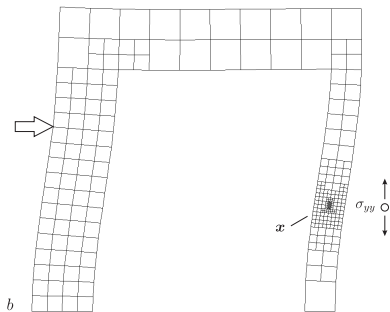
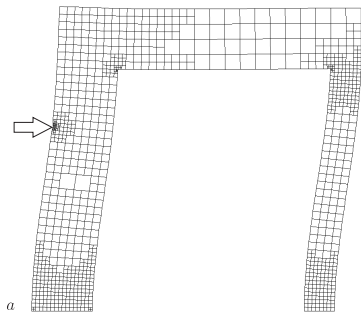
Point values and integral values



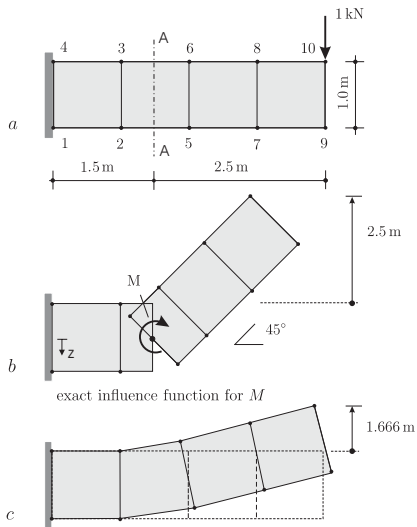
Support reactions are accurate, bending moments are less so.



Goal oriented refinement

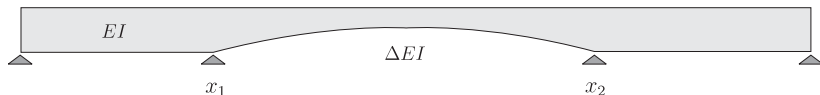


Cantilever plate



FE-approximation on V_h (= bilinear elements)

Model adaptivity



$$\underbrace{\int_0^l EI w'' v'' dx}_{a(w,v)} = \underbrace{\int_0^l p v dx}_{(p,v)} \quad v \in V$$

$$\underbrace{\int_0^l EI w_c'' v'' dx}_{a(w_c,v)} + \underbrace{\int_{x_1}^{x_2} \Delta EI w_c'' v'' dx}_{d(w_c,v)} = \underbrace{\int_0^l p v dx}_{(p,v)} \quad v \in V$$

Zusammenfassung

- ▶ Statik ist Kinematik
- ▶ Finite Elemente = ungenaue Kinematik
- ▶ Einflussfunktionen tragen die Informationen von A nach B
- ▶ Globale Effekte lassen sich lokal voraussagen